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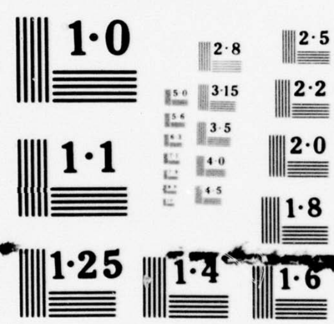

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ANALYTIC METHODS FOR STUDIES OF COMMAND AND CONTROL IN ANTI-SHIP CRUISE MISSILE DEFENSE

Vector Research, Incorporated
Post Office Box 1506
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15 August 1977

Final Report

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Prepared for

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This research focuses on analytic methods to be used to determine the constraints on excess capability of command and control systems in defense against anti-ship cruise missiles. Two approaches were examined. The first, based on the theory of networks of queues, led to the conclusion that further basic research was required. The second, based on the theory of mathematical programming, shows more promise, particularly in applications during the design and development process.

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SUMMARY

This report documents the results of research carried out by Vector Research, Incorporated, with the intent of investigating analytic methods for use in the design and evaluation of command and control systems in engagements between one or more ships and multiple anti-ship cruise missiles. Two approaches were followed. The first involved the theory of networks of queues, and resulted in the conclusion that several basic theoretical developments were necessary to provide a useful analytic methodology. The second approach, based on the theory of mathematical programming, appears to offer more immediate promise in terms of providing bounds on the performance and impact of various command and control hardware and software. The results of both approaches are preliminary and further research and development is recommended.

PREFACE

A major problem currently confronting the Navy is the defense of surface combatants against anti-ship cruise missiles. The problem is difficult and complex primarily because of the numbers and performance of cruise missiles likely to be employed in any attack on a single ship or a task force. A critical issue in responding to the threat is command and control; perhaps more than ever before the command and control system will be stressed and may be the limiting factor in determining engagement outcomes. Analysis and evaluation of command and control is not simple and for the most part detailed Monte Carlo simulations of proposed systems have been utilized to provide information. There is a need for a less expensive methodology for analysis and evaluation, particularly in the design phase, and in any evaluations of proposed designs in which details have not been finalized. The research reported in this document was undertaken in response to this need.

The principal authors of this report are W. Peter Cherry and Robert L. Farrell. The authors wish to acknowledge the contributions of Dr. Ralph L. Disney, who participated in discussions and provided valuable insights and material pertaining to the theory of networks of queues.

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1. INTRODUCTION

This document is the final report on research performed by Vector Research, Incorporated, (VRI) under Contract Number N00014-76-C-0766 with the Office of Naval Research. The intent of the research was to examine the application of the theory of networks of queues to the development of models of command and control or combat direction, particularly in the context of defense against anti-ship cruise missiles. The results of the research are twofold. First, the application of the theory of networks of queues to command and control was investigated and, although useful in certain situations, was found to be not as flexible as desired in the general case. The second area of research grew out of this lack of flexibility. It is based on a mathematical programming analysis of an engagement between one or more ships and multiple anti-ship cruise missiles and shows more promise as an analytic tool for the design and evaluation of command and control systems.

1.1 Anti-ship Cruise Missile Defense.

The evolution of defenses against anti-ship cruise missiles has been characterized by the development of components of the total defensive system. Only recently, for example, in the Shipboard Intermediate Range Combat System (SIRCS) program, has an integrated approach been taken. The problems involved in the defense of a ship against anti-ship cruise missiles are complex and difficult. Essentially a ship may be thought of as a platform which employs guns, missiles, and electronic warfare to destroy, deceive and otherwise defeat cruise missiles. All of these resources are finite in number and do not operate instantaneously.

Similar descriptors apply to the sensor systems, e.g., radar, infrared, electro-optical or electronic, used to detect, acquire and track enemy cruise missiles and decoys, and friendly systems. The threat has been continually growing. Cruise missiles have become much less easy to detect, track and destroy. Maneuver capabilities have increased, as have velocities. Warhead lethalities have grown to the point where probabilities of kill or incapacitation given a hit (or near miss) are high. Coupled with these growths has been the development of tactics which include the simultaneous attack of a single ship by multiple cruise missiles, made possible by the proliferation of launch platforms and improvements in offensive command and control. Finally, offensive electronic warfare has further complicated the problems of conducting a successful defense since sensors can be jammed and deceived, and the emissions of shipboard sensors and systems can be utilized as target information.

The command and control or combat direction system in a defensive engagement performs a number of functions in coordinating and controlling sensors and weapon systems, including:

(a) management and storage of information describing the current *perceived* threat,

(b) assignment and management of resources to update and maintain the currency of the above information,

(c) maintenance of information describing weapon system availability and status,

(d) prediction of future status from current information, and

(e) assignment and reassignment of weapon systems to counter the threat.

The contribution of the command and control system to total defense has grown increasingly critical. Engagement of any single threat weapon system requires that a sequence of events must take place, beginning with detection and ending with kill assessment. Engagement durations are extremely small, and the command and control system must be capable of completing the required sequence of activities very quickly. As noted above, the resources available to the command and control system are finite in number and non-instantaneous in operation; hence the system can become saturated. Finally, the command and control system can be viewed as a decision maker, acting on imprecise information to deal with an uncertain future. Clearly the algorithms used to make decisions are of critical significance to successful defense.

The intent of the above discussion was to illustrate the complexity of the processes of anti-ship cruise missile defense. That the processes are important should be clear. Cruise missiles are inexpensive relative to the costs of the ships they threaten and are being widely deployed on various surface, submarine, airborne, and landbased platforms. As is frequently the case, defensive capability has lagged behind the introduction of this offensive weapon system. The research undertaken by VRI in this project was intended to provide methodology for the evaluation and improvement of the defensive capability.

1.2 Current Methodology. The analysis of the contribution and impact of command and control in defense against anti-ship cruise missiles currently relies heavily on detailed, complex Monte Carlo simulations. These

simulations, for example, the Relative Contribution Model (RCM), or the System Performance Evaluation and Requirements Simulation (SPEARS), are detailed models which essentially "play out" explicitly all the events which comprise an engagement between a surface ship and anti-ship cruise missiles. The representation of the command and control process in such models is facilitated by the fact that in the extreme the algorithms used in the actual process can be made part of the simulation. As with any detailed model or simulation, data requirements are complex and extensive, and data collection thus has a major impact on the resources necessary to perform an analysis or study. In addition, the nature of Monte Carlo simulations has an influence on the manner in which studies are conducted. Essentially the approach must be experimental: by varying input parameter values the analyst determines output changes and then attempts to determine cause-effect relationships. This may or may not be an easy task, depending upon the complexity of the simulation and the degree to which unsuspected relationships exist. Furthermore, although it is usual to replicate a Monte Carlo simulation several times, costs sometimes restrict the number of replications made. As a consequence, outcomes of low probability may not occur. These outcomes may be extremely significant, i.e., although the probability of occurrence is low, the significance may be high. Finally, it can be noted that detailed simulations are best suited for the analysis of detailed systems. In the early stages of top down design, data is not available to support detailed simulation, and it is during this period that major tradeoffs may be made between different subsystems that comprise a defensive system in terms of both numbers and performance.

The experience which led VRI to propose the development of analytic methodology for the representation of the command and control process began with an investigation of methodology used in anti-ship missile defense studies.[1] It was first noted that because of the complexity and costs associated with the simulations used, analysis of multiple-ship engagements, tactics and doctrine, and capability was constrained. Accordingly, the development of an analytic methodology was initiated to provide a more efficient analysis tool.[2] In the course of developing this methodology it was recognized that in order to give an adequate treatment of command and control further research was necessary. This led to the research described in this report. The value of the effort lies in two areas. First, it is directed to providing a useful and efficient tool for analysis of fleet anti-ship missile defense. Second, it will, because of reduced complexity and data requirements, provide an analysis methodology useful in the evaluation and comparison of concepts in the early stages of system development.

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[1]Bonder, S., Cherry, W. P., and Miller, J. M., Development of Analytic Methodology for Naval Planning Areas, VRI report number ONR-1 FR73-1, Vector Research, Incorporated, Ann Arbor, Michigan, 1973.

[2]Cherry, W. P., Farrell, R. L., Miller, J. M., and Moore, M. H., A Single Ship/Multiple Cruise Missile Engagement Model for Fleet Air Defense Planning, VRI report number ONR-1, FR75-1, Vector Research, Incorporated, Ann Arbor, Michigan, 1975.

1.3 Outline. The remainder of this report is organized as follows. Chapter 2.0 describes a mathematical programming approach to modeling the command and control process in anti-ship cruise missile defense, while chapter 3.0 discusses the application of the theory of networks of queues to the same problem.

2. DESIGN CONSTRAINT ANALYSIS AS A TOOL FOR COMMAND/CONTROL SYSTEM STUDIES.

In the process of analyzing the performance of a complex weapon system against potential enemy forces, there are many kinds of analyses which prove useful or necessary. Some of these concentrate on the physical performance of such components of the system as the sensors, air frames, or warheads of the overall system. Such analyses serve two major purposes -- they aid systems designers working on the details of the individual components, and they provide data for designers of one or another component or subsystem about the expected performance of other particular components. Such analyses and the detailed models and experiments which are associated with them cannot be omitted from the development program of any complex weapon system.

On the other hand, an aggregate of such analyses and models is not sufficient to completely address the design of complex weapon systems, particularly those involving significant elements of command and control. In such complex systems, it is not merely the performance of the individual parts which must be optimized, but the portions that connect and interface the components: that is, the command and control subsystem, among others. One approach to the analysis of alternate command and control subsystem designs is to simulate or experiement with the entire system. In this approach, the basic component analyses and models are drawn upon to provide simplified logic to represent the probabilistic performance of the various other components, and a detailed experiment, exercise, or Monte Carlo simulation is constructed using these and a detailed description of the

command/control alternative considered. The resulting model can be exercised for a number of replications to provide a statistical estimate of the overall system performance. This operation can be repeated with alternative designs for the command and control portions of the system, and some selection made among alternatives.

Some such process is a necessary part of the development of any complex weapon system with significant command and control elements. However, additional aids would be useful to help system designers and specifiers in two areas where the current approach has major limitations. First, while experimentation of simulation allows for the comparison of command and control alternatives, it does not provide for a clear answer to the question, "How close is this system to the best achievable performance?" The performance of a command and control system must, indeed, be rated in terms of the performance which it manages to achieve from the entire system. This supplies a method of measuring performance on an absolute scale. However, other system components limit the performance which <u>any</u> command and control system could achieve, and it is often of critical importance to recognize whether one is at or near this limit.

When a system is in the region of this limit, only changes in the numbers of subsystems or in the design of the other components can be expected to pay off in system performance improvements. No further improvements in command and control elements can be expected to be significant without other changes, and if other changes are made, command and control changes may be unnecessary. Recognition of the potential

limits on system performance which are imposed by non-command/control elements is critical not only in order to recognize when further design work on the command and control system will not pay off, but also in order to determine the measures to use and the threat and environment in which to evaluate in detail various command and control alternatives. The differences in performance between alternatives will show up most plainly in the region near, but clearly below, the maximum feasible performance region: appropriate measurements conducted in this region stressing differences in performance will most clearly differentiate command and control designs. Examination of alternatives in the region which cannot be achieved by any alternative will normally show little difference between systems, since all will fail to meet the design criteria. On the other hand, examination in regions far below the critical limits will not differentiate systems clearly enough, since many or most designs may be expected to perform quite well, and therefore nearly equally.

In addition to this need for tools with which to determine where the performance limits of command and control systems really are, designers need less detailed, less expensive tools to guide their early design efforts. These efforts are typically conducted without a specification of a command and control system in adequate detail to simulate exactly that system and no other, and with little information available on the values which may be achieved by stresses on various qualitative design criteria. Extensive experimentation or simulation is often financially unfeasible, and the necessary decisions are therefore taken with little or no analysis support.

This chapter briefly describes an approach to the analysis of command and control system performance which might help to fill these two present voids. It is described for the specific problem of a naval air-defense command and control system, but appears to the authors, at least, to have some promise for application to other problems.

The method, which is a type of mathematical analysis, involves no truly new mathematics, although the application of the relevant mathematics to these problems seems to be new. It is not intended that it could supplant Monte Carlo simulation or experimentation, but rather to supplement them in the areas described above. Because it is more a method of using mathematical models and analysis than a specific model or model solution technique, the exposition consists of two major parts: a description of the development of and examples of potential uses for a set of mathematical models for a particular applications area, and an intertwined discussion of the motivation and intentions behind the developments, in order to convey more clearly how parallel problems might be attacked in analogous ways. Of course, in a real application, the first steps in any analysis involve determing the measures of effectiveness and combat situations of interest. Since this is only a methodological exploration, less time was spent on this step than would normally be taken, and no significant exposition will be required. The setting which was chosen for our investigations involved a parametrically variable number of threat cruise missiles attacking one or more high value targets. Additional cruise missiles may be added, attacking other targets, as may decoys, without substantial changes in the methodology. The

initial detection times at which the missiles are detected are parametrically variable, as are the flight details and the times at which they will impact a high-value target. For our demonstration of the analysis techniques, we have taken all the cruise missiles to be similar, although multiple types can be analyzed. The defense forces have one or more defense weapons which may be used against these missiles: each weapon may have different performance, and the periods (if any) during which each weapon may fire at any particular missile may be different, although the numerical examples which will be given and many of the special cases for which particularly simple analytic results will be discussed will involve similar weapons.

Given this overall situation, the measure of effectiveness of the defense system <u>in toto</u> is taken, for our demonstration, to be the probability that any cruise missile impacts a high-value target. This choice makes the various analytic forms simpler while still managing to illustrate the basic approach. A similar analysis could have been conducted using the probability that N missiles reach their target, the mean number of missiles reaching their targets, or some other potential measure.

As will be seen in the discussion proper, the essential nature of the approach requires a single measure of effectiveness for any single analysis step. Thus, this method offers no new approaches to the problems inherent in the existence of multiple measures of effectiveness. Problems in which such multiple measures exist may be treated by separate analyses of each measure and by use of composite measures constructed in any of the available ways.

For the demonstration, certain real details of the processes were neglected, although they could be included without difficulty at the cost of complicating the exposition and clarity and simplicity of the results. These include such items as reliability and operational failures, electronic warfare, or other environmental variations during a single combat engagement, repeated engagements, etc. Since the measure of performance of the entire system is the probability that any one missile reaches its target (or, in a positive, rather than a negative sense, the probability that no missile reaches its target), we consider the performance of the command and control system to be measured in these terms also. This is central to the approach being described: the performance of the command and control system is measured in operational terms, not in engineering terms. Its effectiveness is considered to be the effectiveness which it can bring out of the system as a whole, no matter what its internal characteristics.

Given this as the measure of effectiveness of the command and control system, we may now turn to the first of our two questions: "How well could a perfect command and control system do with the available resources?" This is naturally viewed as a constrained optimization problem. The quantity to be minimized is in our setting the probability that at least one missile reaches its target, and in the general setting is whatever measure of effectiveness is of interest. The constraints are the specifications of the threat and the performance parameters. The space within the constraints represents the feasible command and control system performances.

Without turning immediately to the problems involved in trying to solve such a problem, let us look at our second question in this setting: "What are the comparative values of different design approaches or design details to the command and control subsystem?" In the same way that we can view the first question as posing a constrained optimization problem for solution, this question asks the sensitivity of the solution of that problem to changes in the constraints. That is, we constrain the system design not only by the threat and physical performance parameters but by various restrictions on the design (e.g., consider cases in which defensive systems are uncoordinated in their fires, coordinated as to sector but not target, or potentially fully coordinated; or consider systems in which the number of targets being tracked simultaneously is limited; etc.), and ask how the optimal performance of systems constrained in one such way compares with the optimal performance of systems constrained in another.

In this approach, the usual view of a system design as a point in some design space is altered slightly so that a system design point is viewed as the limit of some set of design neighborhoods corresponding to specific kinds of constraints introduced by particular design decisions. In most applications which we have been able to envision, these are simply different views of a single topological design space: one a pointwise view generally taken through a coordinatization of the space, and the other a direct view of the topology. It should also be noted that we think of the constraints as specified by the feasible region, rather than as parametric surfaces in their own right. Again, these are mathematically equivalent views.

The practical problems in application of this view of the problem are, of course, appropriate parametrization of the design choices and constraints. In order to explore the practical feasibility of the approach, we will first describe a group of general applications clustered around the problem setting described earlier, including both analytic and numeric detailed examples. The results here should be taken only as an initial indication of possible fruitfulness in the general approach of design constraint analysis of command control system performance. The actual value of these techniques can, of course, only be analyzed with further developments and actual applications of them.

Consider first, the analysis of the maximum performance of a command and control system which has available N defensive missiles with (constant) kill probabilities P_k.

Theorem 1. The performance of such a system is (tightly) bound by F (N; P_k, M) where F is the negative binomial distribution function and M is the number of threat missiles.

The proof is straightforward, once a single element is observed. This element of technique, which is most useful in almost all the example analyses, is to take an inverse view of the killing process and consider not the stochastic process of kills by defensive weapons, but the counting process which describes defensive weapons required to kill the threat missiles. For the (fairly normal) constant P_k cases, each threat missile requires a geometrically distributed number of defensive rounds

fired at it to kill it. Since the sum of N geometrically distributed random numbers is negatively binomially distributed, the result is immediate.

In order to make clear the practicality of these methods, a small example will be carried through numerically. Although this example is small and simple -- so that the associated calculations have been performed on a hand calculator -- large-scale problems can be treated on a computer in tenths of seconds.

Consider a system with ten available defensive "rounds" expecting to be engaged by 3 to 5 threat missiles. Take P_k = .5 or .6. Table 1 shows the performance limits of the system.

TABLE 1. EXAMPLE SYSTEM EFFECTIVENESS

Threat (M) \ P_k	.5	.6
3	.945	.988
4	.828	.945
5	.623	.834

Now let us ask what the maximum performance of our system would be if it were constrained (by either physical or command and control design) to fire defensive ordnance no more frequently than every Δ time units.

In order to solve this problem, we need a bit more notation: Let $T_1 \leq \ldots \leq T_i \leq \ldots \leq T_M$ be the impact times of the threat missiles, should they survive the defenses. Let $T_0 < T_1$ be the time they are first detected. (Our example theorem considers simultaneous detections; the extension to different detection times follows the same general pattern of analysis.)

Theorem 2. Let G be the geometric probability distribution with parameter P_k. Let H_n be the (incomplete) probability distribution defined as follows

$$H_1 = G, \text{ truncated to max } ([T_1/\Delta],N)$$

$$H_i = H_{i-1} \circ G, \text{ truncated to max } ([T_i/\Delta],N)$$

where [x] denotes the greatest integer in x and o denotes convolution. Then our maximum system performance under these constraints is $H_M(\max([T_M/\Delta],N))$.

The proof is straightforward. We are simply solving, for the probability of the effective system region under a multiple, independent geometrically distributed rounds to kill assumption. The recursion is the simplest formulation of this, but it should be noted that this is merely a convenience: the underlying structure is the evaluation of the probability of a region under a known joint probability density.

Using the same data as before, but now assuming that two threat missiles will impact at 5Δ and the remainder at 10Δ, our performance limits are shown in table 2. Alternative threat arrival patterns would lead to different results, and in actual cases extensive sensitivity analysis would probably be done on this variablity.

TABLE 2. EXAMPLE SYSTEM EFFECTIVENESS

Threat (M) \ P_k	.5	.6
3	.803	.909
4	.744	.888
5	.592	.806

What happens if we must allocate our fires at a single point in time? Then we are choosing a number of rounds (or a maximum number of rounds) which may be fired at each missile before the results of the first firing are known. We will examine this as an alternative to the rate of action constraint, although it would also be treated as an additional constraint.

Theorem 3. Let R = [N/M]. Then our optimal performance is

$$(1 - P_k^{R+1})^{(\frac{N}{M} - R)M} \quad (1 - P_k^{R})^{\{M - (\frac{N}{M} - R)M\}}$$

The proof follows the previous techniques. In this case, our numerical case gives the results shown in table 3. As the reader can see, in the cases shown, a command and control system which would require significant decision making time, but make its decisions sequentially would have better performance than one which made an immediate, total decision requiring less time. These results are, of course, not intended to be a realistic attack on an actual, presumably much more complex, command and control problem, but only to show the general type of results which can be obtained through design constraint analysis.

TABLE 3. EXAMPLE SYSTEM EFFECTIVENESS

Threat (M) \ P_k	.5	.6
3	.628	.854
4	.421	.618
5	.237	.418

The initial experiements with design constraint analysis reported above appear to indicate that it may have a potential for providing a useful aid in the study of command control systems. This potential possibility suggests that further research pursuing these ideas in a specific project setting might both assist the specific project and provide more definite information on the overall value of the design constraint analysis approach.

3. QUEUING NETWORK APPROACHES TO MODELING COMMAND AND CONTROL

As described in the first chapter, the command and control or combat direction system performs a number of functions in coordinating and controlling sensors and weapon systems, including:

(a) management and storage of information describing the current perceived threat,

(b) assignment and management of resources to update and maintain the currency of the above information,

(c) maintenance of information describing weapon system availability and status,

(d) prediction of future threat status from current information, and

(e) assignment and reassignment of weapon systems to counter the threat.

If the elements (targets) of the perceived threat are thought of as customers, and the resources used to detect, track and counter the targets are thought of as servers, then the defense of a ship is analogous to a stochastic service system. If one further thinks of certain targets waiting for service, e.g. for a track update or for weapon assignment, then the stochastic service system is analogous to a network of queues and servers, i.e., the status of the defense can be described by the numbers of targets awaiting specified service from designated servers, the numbers of targets currently being served, and the status of the servers providing service. The flow of targets through the system corresponds to the dynamic progress of the engagement. The analogy is illustrated in figure 1.

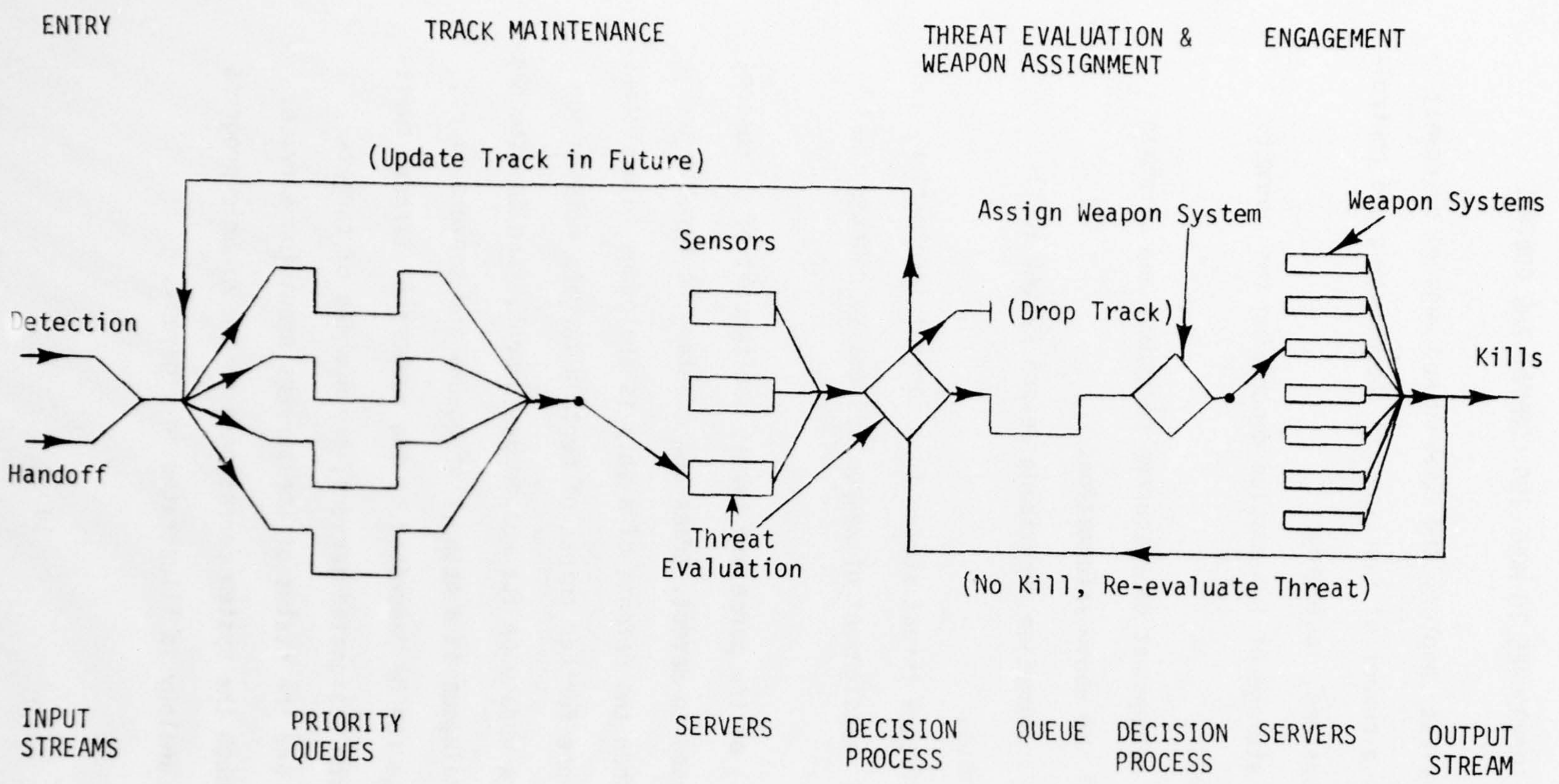

Figure 1. A Queuing Network Representation of an Elementary Command and Control System.

The original intent of this research was to apply the results of theory of networks of queues to the overall command and control process described above. That intent was motivated by the fact that one of the most significant problems encountered in reality was that the storage allocated for the queues of targets awaiting service became saturated, and the command and control system in this situation did not degrade gracefully. During the course of the research, the authors became aware of new developments, principally in the field of automatic data processing, that will alleviate this problem, i.e. sufficient storage capacity will exist. The principal problems now appear to lie in the facts that service cannot be provided at a sufficiently fast rate to defeat multiple targets, and that coordination of multiple resources (radars, guns, missiles, etc.) is difficult and complex. The relationship of these problems to the theory of networks of queues is discussed in the remainder of this chapter.

3.1 The Theory of Networks of Queues. The development of the theory of networks of queues has focuses primarily on decomposition of the networks into segments for which steady-state or equilibrium results have been obtained. These results include the impact of stochastic switches acting on input or output streams [Cinlar, 1965], superposition of input or output streams [Cherry, 1972] and feedback [Davignon, 1974]. In these derivations attention has been generally focused on characterizing the stochastic point processes which describe the outputs of servers under equilibrium or steady-state conditions. The application of the theory to more complex networks then proceeds by utilizing the characterization of outputs

of segments of the networks to describe the inputs to subsequent segments. Unfortunately the characterizations, except in those cases where all random variables are independent and negative exponentially distributed, grow increasingly complex and computationally difficult. Another drawback of the theory as it currently exists, in the context of analyzing the command and control process in anti-ship missile defense, is that few if any transient results exist. One may argue that a command and control system is *never* in equilibrium. Instead it undergoes intermediate periods of high demand for service and it is the transient behavior that is most significant. Furthermore transient results are generally more complex than corresponding equilibrium or steady-state results.

3.2 Reneging. A second aspect of the current state-of-the art in the theory of networks of queues that is significant in the context of the problem of anti-ship missile defense is the absence of major results dealing with reneging. A reneging customer in queuing theory is one that departs from the system while awaiting service or during service. In the context of anti-ship missile defense a reneging customer is a missile that impacts upon the ship (or in the multiple-ship case is handed-off to another ship). Considering the case of single ship defense, the problem is to provide service so as to ensure that no customer reneges.

The classic reneging problem is concerned with a birth-death queuing process wherein the death rates are assumed to be of two kinds:

(a) the usual death rates representing rates of service completion and

(b) a defection or reneging rate, $r(N)$, depending on the queue length and representing the rate at which customers leave the queue (defect or renege) before obtaining service.

A model for the above purpose was originally posed by Barrer and called "customer impatience". Gnedenko and Kovalenko extended this version considerably and we will discuss their model. It is stated in simplest form as follows:

Let us assume a service system with n identical servers. A Poisson process (λ) arrives to this system. Service times are identically distributed and independent negative exponential random variables (ν). Any arrival is either immediately served or waits his turn. The waiting, however, is limited by some τ. If the customer's service has not started within τ after his arrival, he is lost.

The main characteristic of interest is the probability of loss. Two cases are considered initially:

(a) τ = constant

(b) τ is an exponentially distributed random variable.

Barrer defended his model in a military application by saying, "An attacking airplane engaged by antiaircraft or guided missiles is available for 'service', i.e., within range, for only a limited time." Presumably, if the attacker is not satisfactorily served (i.e., "killed"), his leaving of the queue can be construed as an attack on the defender. Therefore, in the overall structure one would like to construct a server whose service rates are sufficiently fast (and accurate if one adds a condition that service may not be satisfactorily

concluded and must be repeated with some probability) so that the probability of a customer having the queue is small. That is, this class of models must ultimately be concerned with an optimization problem. There do not appear to be any published results concerning the optimization stage for this class of queues.

The queuing problem of Barrer and Gnedenko and Kovalenko has been studied as follows: For the τ = constant case of Barrer, one can derive a system of partial differential, difference, integral equations. These equations define the transient response of the vector valued Markov process $\{\xi|\tau), N(t)\}$. $N(t)$ is the number in the queuing system at t. $\xi(t)$ is a random vector whose elements $\xi_i(t)$ are defined to be the length of the interval after t until the j^{th} server finishes serving customers arriving before t. $\xi(t)$ is of dimension n. In effect, the Markov process $\{\xi(t), N(t)\}$ is an $(n + 1)$ dimensional, continuous parameter process. The given equations determine the transient behavior of the "state" probabilities. A state is an $(n + 1)$ triple $(k, x_1, x_2 \ldots x_n)$ with k integer valued and x_i a non-negative real number. No properties are given for the solution to these equations. A study has been made of some of the asymptotic properties. It is unlikely that, except for small values of n, one will have much hope of solving these equations explicitly even in the exponential server case such as Barrer considered. A steady state solution is obtained by Gnedenko and Kovalenko. Dimensionality problems will probably preclude numerical solutions to problems of moderate size.

The problem is considerably simpler when the bound on the waiting time τ is a random variable with an exponential distribution. Then the queue length process $\{N(t)\}$ is Markov and is a birth-death process. While

transient solutions are not obtained, the steady state solutions are immediately available and are given. Transient solutions may be available if n is not too large.

In summary, this Barrer-like model with limited waiting times appears to be an enormously difficult model to analyze for transient response of the state probabilities.

A generalization to the model is provided by Gnedenko and Kovalenko wherein the limitation is on the total time the customer can spend in the system. τ then is this maximum time allowed and can be either fixed or random. In this model the customer can leave the queue (whenever τ is exceeded), leave the server (whenever queuing time plus service time spent exceeds τ), or be served to completion (whenever waiting time plus service time is less than τ). Service times are now general so we are looking at an M/G/n queue with defections. As previously, the vector-valued Markov process $\{\xi(t), N(t)\}$ is explored. However, unlike the previous case, no transient analysis is made. Most properties discussed are for the $\{\xi(t)\}$ process only. These authors immediately jump to steady state properties. Even here, the equations governing the steady state probabilities appear to be rather complicated integral equations that would have to be explored numerically. Steady state results for $\{\xi(t)\}$ are known for the same two cases as previously. There does not appear to be any other available results.

Both of the foregoing models are further studied by allowing τ to be random but not necessarily exponentially distributed. Results here are meager. Only the single server case is studied and only the $\{\xi(t)\}$ process

is discussed. Theorems are given to guarantee the existence of steady state behavior, but except for the previous (above) special cases little is done with these results.

In summary, there are several more or less developed models for queues with limited holding times. Except for the simplest cases, nothing seems to be known about transient responses and the steady state results, when they exist, they appear to involve difficulties for computing except when τ is an exponentially distributed random variable. The problems are inherent in the requirements that service times be of the GI form and the limitations imposed. These assumptions require that one always carry along "status" in the $\xi(t)$ process which then produces a large vector valued Markov process in the time residual service time space necessitating the study of large scale integro-differential, difference equations, or complicated integral equations.

General transient studies of these systems may be impossible numerically, for any but the simplest cases (e.g., for 2 servers of "nearly" exponential type). Unfortunately, one does not escape from these problems even when steady state problems are studied. Consequently, a detailed model to produce numbers would have to be built. Whether one could get approximate results by numerical methods or by using various diffusion or heavy traffic models would be an avenue of further research. (These diffusion models or heavy traffic models would only make sense for a study of the $\{\xi(t), N(t)\}$ process. They would be meaningless for the $\{\xi(t)\}$ process alone.) None of these models include "feedback" in the structure, which could occur if kill probabilities were not 1.

3.3 Priority Queues. The problem of modeling the allocation of resources in queuing networks has focused primarily on the theory of stochastic switches which direct customers to queues or servers in the network following completion of service by a specified server. The stochastic behavior of such switches has been studied for a variety of disciplines, including those that depend upon customer type, primarily in a manner that can be described by a Markov process. It was the intent of this project to apply the theory of these switches to modeling the threat evaluation and weapon assignment process, utilizing the theory of priority queues, i.e., cruise missiles would be assigned a priority upon detection and service would be provided in accordance with the mix of priorities present at any given time.

The general approach to priority queues is to assume that a customer upon arrival to the queuing system is assigned a parameter or parameters that determine his relative position in the queue for service. Most of the results obtained for priority queues deal with a finite set of priorities which do not change during the period in which the customer is present in the system. Limited results exist for continuous parameter priorities and for dynamic priorities, i.e., those which depend upon the length of time that a customer has been present in the system. In the context of the defense of a ship against anti-ship cruise missiles, it is the latter dynamic case that is of interest since the threat posed by an incoming cruise missile continually grows as a function of time, i.e., as the missile gets progressively closer to the ship.

The principal results in the area of dynamic or time-dependent priorities are discussed by Kleinrock. Kleinrock's model assumes that upon arrivel to an M/G/1 system, customers are assigned one of a finite number of priorities b_p where $o \leq b_1 \leq \cdots \leq b_P$. If the instant of arrival is denoted by τ, at any time t subsequent to τ the customer's priority is given by:

$$q_p(t) = (t - \tau)b_p$$

Kleinrock carries out an analysis of this system for the case of exponential service times and obtains the expected waiting time of a customer of type p as well as the expected numbers of customers of various priorities in the system, both under equilibrium conditions. He extends these results to consider time dependent priorities of the form:

$$q_p(t) = (t - \tau)^r b_p$$

again obtaining expected waiting times under equilibrium conditions, and studies the behavior of the system as r tends to limits zero or infinity.

As in the case of queues with reneging, transient results are unavailable for priority queues, and the absence of the results has a significant impact on the usefulness of the theory as a tool for analysis of command and control in anti-ship missile defense. Furthermore, the results have been obtained under the assumption that all customers remain in the system until service is completed. Some results due to Rao are given in Jaiswal for preemptive-resume priorities with balking and reneging, but these deal again with expected values under equilibrium in this case of the busy period and the joint queue length probabilities. Even in the equilibrium case, the computations required are not simple.

3.4 Conclusions. In its simplest form, the defense of a ship against anti-ship cruise missiles can be restated as follows. The sensor systems of the ship are employed to detect incoming missiles and to subsequently provide an extimate of their velocities. These estimates of velocity can be used to determine the length of time during which weapon systems can be employed to defeat the incoming missile; if this is not accomplished during the length of time available, the missile will impact and catastrophic consequences can be assumed. The length of time is a random variable corresponding to the reneging time referred to in section 3.2 of this chapter. Its randomness arises from the fact that estimates of position and velocity are imprecise and from unknown variations in cruise missile course due to deliberate or random causes. It can be argued that the estimate forms the basis of any threat evaluation procedure and in turn leads to the assignment of a priority to the incoming missile, i.e., the relative threat posed by the missile. This priority increases as a function of time and the mix of priorities changes as missiles are detected, destroyed, or as destruction is anticipated because of weapon assignment. Threat evaluation and weapon assignment algorithms thus operate on estimates of time available and weapon system performance and assign priorities which grow as a function of elapsed time. A critical question is therefore: given the performance of the sensors and weapon systems of a ship, what is the contribution of command and control or combat direction to the defensive capability of the ship as an integrated system, and what is the sensitivity of that capability to variations in numbers and behavior of anti-ship cruise missiles?

In addressing this question, the project staff concluded that several developments in queuing theory were required including:

(a) a model that focused on descriptions of transient behavior,

(b) a model that included both reneging and time-dependent priorities for service, and

(c) computationally tractable methods for modeling the dependencies between different segments of networks of queues, particularly those in which random variables do not have negative exponential distributions.

As research in these areas was undertaken, it was concluded that each of these topics would require a major effort not consistent with the original scope of the project. This led to the approach based on the theory of mathematical programming described in the previous chapter. Although the project staff is of the opinion that further effort aimed at enriching the theory of queueing networks would be beneficial, the staff feels that the mathematical programming approach offers a high payoff at this time.

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